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Wormholes and the problems of the Time Machine Construction

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In 1949 Kurt Gödel opened us the theoretical principle of the Time Machine Consruction. But the practical questions of realization of this theoretical possibility require to solve a number of problems.

1. A natural Time Machine in simply-connected space-time can exist only in extremal conditions.

Let's assume that the closed time-like smooth curve L is an analytical Jorgan's curve and one lies on an simply-connectedt surface $F \subset D$, L is border of F, and L is contained in the space which is filled a dust matter with density ρ.

Then the Zelmanov's chronometric invariant time $\tau(L)$ of living among the world line L can be estimate as it follows:

$$\tau(L) = \frac{1}{c}\int\limits_L \frac{g_{0i}dx^i}{\sqrt{g_{00}}} \sim \frac{\sqrt{8\pi G\rho}}{c^2}\sigma(F), \quad \sigma(F) = \iint\limits_F dS \tag{1}$$

From (1), if we allow "Euclidean" relation $\sigma(F) \sim \pi^{-1}[l(L)]^2$, where

$$l(L) = \int\limits_L \sum_{i,k=1}^{3} \sqrt{\left(-g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}\right) dx^i dx^k}$$

is spatial length of loop L and $\sigma(F)$ is "Euclidean" area of a surface of F, it follows that

$$\tau(L) \sim 2 \cdot 10^{-24}\sqrt{\rho} \cdot [l(L)]^2 (sec).$$

From this formula it is visible that causal chains exist or in extremal physical conditions, or have the sizes of galactic scale [1].

2. The Thorn's Time Machine Construction is impossible [2,3,4]. Thorn and others declare that closed timelike curves may arise as a result of the relative motion of the 3-wormhole's mouths. The Thorn's Time Machine construction is based on the Lorentz contraction [5,6] or on placement of one mouth of a 3-dimensional wormhole in a strong gravitational field [7]. But

1) the formulas of the Lorentz transformations were deduced only for the simply connected domain of 4-dimensional arithmetical space $\mathbb{R}^4$ [8]. So the using of concept of Lorentz contraction for nonsimply connected space-time is not correct;

2) we must consider the two pairs of clocks: external t_A^{ex} and internal t_A^{in} clocks near left mouth A of wormhole, and external t_B^{ex} and internal t_B^{in} clocks near right mouth B (see Fig.1). Always clocks t_A^{ex} and t_A^{in} are synchronous, and clocks t_B^{ex} and t_B^{in} are synchronous, because they are located in the same physical conditions. Hence, the clocks t_A^{in} and t_B^{in} can not be synchronous, and transition from mouth B to A through wormhole is not an exit in the past.

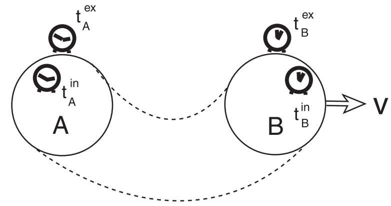

Fig.1.

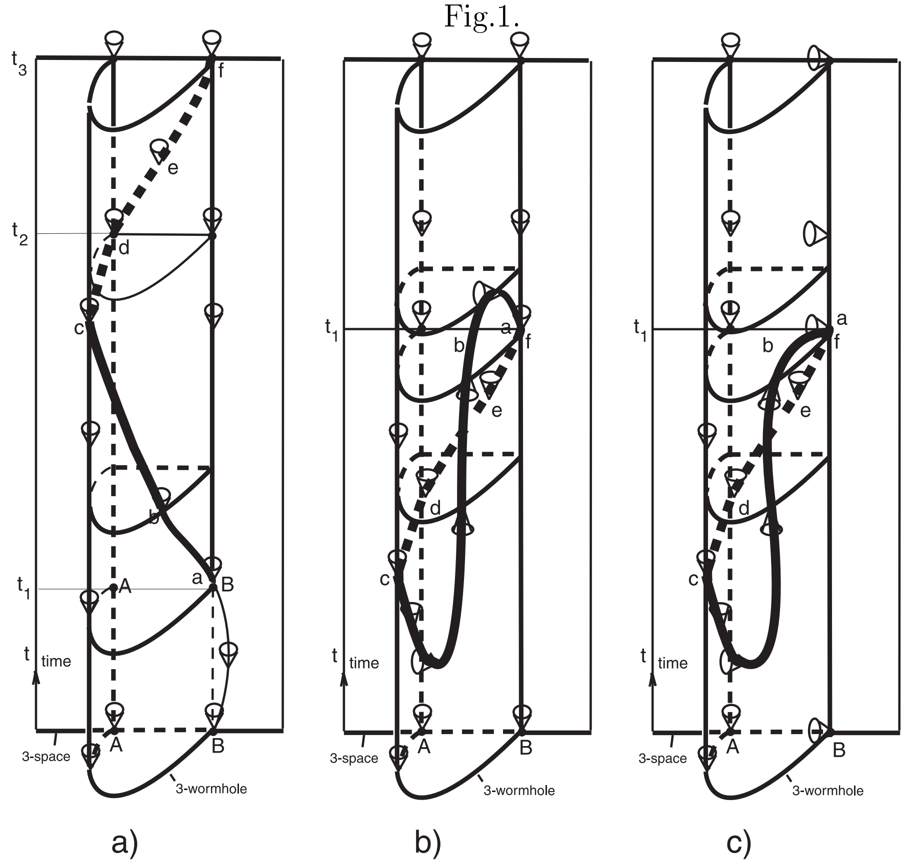

Fig. 2.

3) the time-like loop appears as result of an aprioristic declination of the light cones. In the Thorn's time machine the light cones in space-time are set initially. Therefore, if initially necessary declination of light cones in space-time for existence of time-like loop was absent, than the time-like loop won't appear, when mouth B is moving there-here, or the wormhole's mouth B is placed in the strong gravitational field. And then a transition through wormholes from mouth B (the enter) to mouth (the exit) A (a curve of $abcd$ for Fig. 2)) and the next movement from mouth A to mouth B (the curve of def for Fig. 2)) will lead only to the exit in future epoch $t = t_3$ with respect to the beginning of all movement among the curve $abcdef$ at epoch $t = t_1$ (see Fig. 2, a)). Let's turn attention to that an declination of "in" and "ex" cones of the wormhole at each mouth shouldn't differ strongly. Moreover, it should not differ strongly for both mouths, otherwise the time flows on the mouth-ends differently, and it is senseless to speak about any synchronization of the clocks along a wormhole in its internal space.

The unique hope that in a wormhole the light cones cardinally change the orientation. Really, at Fig. 2, b) such situation containing an time-like loop is represented. But it is absolutely impossible, that in internal space of wormhole having length in, say, 1 meter, such somersaults of a light cone are possible.

If mouth B lies in strong gravitational field, then we have the same situation with somersaults of a light cone (see Fig. 2, c));

4) all known to us the examples of 3-dimensional wormholes are metrics on the cylinder

without necessary somersaults of light cone. The cylinder is not a space with a 3-wormhole, and the declaration that the cylinder easily transforms to space with 3-wormhole by means of obvious identifications of points (and the Lorentz contraction is observed) is only the doubtful declaration.

3. The Time Machine Construction is 4-dimensional wormhole which connects two events (at present and at past) after the transformation of space-time M^4 into a resilient leaf (or dense one) in 5-dimensional Hyperspace M^5.

In [2] we suggested another project of time machine using a 4-dimensional wormhole.

Let $< M^4, g_{\alpha\beta} >$ be a leaf of an orientable foliation $\mathcal{F}$ of codimension 1 in the 5-dimensional Lorentz manifold $< M^5, g^{(5)}_{AB} >$, $g = g^{(5)} \mid_{M^4}$, $A, B = 0,1,2,3,5$. Foliation $\mathcal{F}$ is defined by the differential 1-form $\gamma = \gamma_A dx^A$. If the Godbillon-Vey class $GV(\mathcal{F}) \neq 0$ then the foliation $\mathcal{F}$ has a resilient leaves.

We suppose that real global space-time M^4 is a resilient one, i.e. is a resilient leaf of some foliation $\mathcal{F}$. Hence there exists an arbitrarily small neighborhood $U_a \subset V^5$ of the event $a \in M^4$ such that $U_a \cap M^4$ consists of at least two connected components U^1_a and U^2_a.

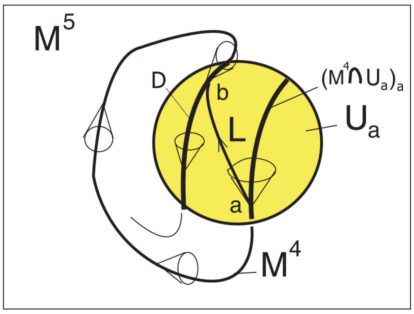

Fig. 3.

Remove the 4-dimensional balls $B_a \subset U^1_a, B_b \subset U^2_a$, where an event $b \in U^2_a$, and join the boundaries of formed two holes by means of 4-dimensional cylinder. As result we have a 4-wormhole C, which is a Time machine if b belongs to the past of event a (see Fig. 3.). The past of a is lying arbitrarily nearly. The distant Past is more accessible than the near Past. A movement along 5th coordinate (in the direction γ^A) gives the infinite piercing of space-time M^4 at the points of Past and Future. It is the property of a resilient leaf.

If σ is the characteristic 2-dimensional section of the 3-dimensional domain D_0 that one contains the 4-wormhole, than we have for the mean value of energy density jump which one is required for creation of 4-wormhole C the following formula:

$$< \delta\varepsilon > \sim \frac{c^4}{4\pi G} \frac{1}{\sigma},$$

where c is the light velocity, G is the gravitational constant.

When does a foliation have a spring leaf? For example, if $\mathcal{F}$ be a codimension one transversely oriented, transversely affine foliation on a closed manifold, then affine foliation cannot have a medium complexity —- it is either so complicated as to contain resilient leaves or so simple as to be almost without holonomy [9].

If foliation $\mathcal{F}$ has no a resilient leaf we transform $\mathcal{F}$ into foliation $\mathcal{F}'$ with resilient leaves with the help of non-integrable deformation $\mathcal{F}_t$, $t \in [0,1]$, $\mathcal{F}_0 = \mathcal{F}$, $\mathcal{F}_1 = \mathcal{F}'$.

The value of energy density jump that one need for this deformation $\mathcal{F} \to \mathcal{F}'$ (with $g^{(5)}_{AB} \to (g')^{(5)}_{AB}$) is equal to

$$\delta\epsilon \sim \frac{\pi c^4}{G} \left[\frac{l(\xi')}{vol'(M^5)}[-2\beta'_1(M^5) + \beta'_2(M^5)] - \frac{l(\xi)}{vol(M^5)}[-2\beta_1(M^5) + \beta_2(M^5)] \right],$$

where $\beta_i(M^5)$ are the Betti's numbers, $l(\xi)$ is the trajectory length of some vector field ξ on M^5.

We can declare that our local power actions in space-time are capable to reconstruct its placement in Hyperspace.

The Gidbillon-Vey class is characteristic class of foliation, which is connected with scalar and electromagnetic fields. In the case of foliation of codimension 3 the characteristic classes are connected with electromagnetic field A_i and gluons $A(3)_i$ and $A(9)_i$.

4. Using of dense leaves. Another method of the time travel is an using of the dense leaves. If M^4 is a dense leaf in $\mathcal{F}$, then in a dense leaf there is there is a possibility to make transition in the past, having left in Hyperspace and having passed rather small distance. It is a question: in what moment and from what point of dense leaf such trip is possible? But we see that the possibility of such travel exists. If all leaves of the foliation are dense, i.e. the foliation is minimal one, than the travel to the past is possible from of any leaf.

5. Conclusion. We see that construction of the Time Machine requires the solutions a number of geometrical problems of the foliation theory.

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