Proceedings of the Eighth Marcel Grossmann Meeting
on General Relativity, 22-27 June, 1997, The Hebrew University,
Jerusalem. Singapore: World Scientific, 1999.

TIME MACHINE AND FOLIATIONS

A.K. GUTS

The following mechanism of action of Time machine is considered. Let space-time $< V^4, g_{ik} >$ be a resilient leaf of a foliation $\mathcal{F}$ of codimension 1 in 5-dimensional Lorentz manifold $< V^5, G_{AB} >$. Hence there exists an arbitrarily small neighborhood $U_a \subset V^5$ of the event $a \in V^4$ such that $U_a \cap V^4$ consists of at least two connected components U_a^1 and U_a^2. Remove the four-dimensional balls $B_a \subset U_a^1, B_b \subset U_a^2$, where an event $b \in U_a^2$, and join the boundaries of formed two holes by means of 4-dimensional cylinder. As result we have a four-dimensional wormhole C, which is a Time machine if b belongs to the past of event a.

We have a Time machine in a space-time domain D, when a smooth closed time-like curve exists in this domain. In the paper [1] the case of creation of Time machine from the three-dimensional wormhole (3-wormhole) by means of kinematic procedures with one mouth of wormhole is considered. The author agrees with opinion of M. Yu. Konstantinov [2] that this assertion is erroneously because contradict to Principle of equivalence.

In this paper the different principle of action of the Time machine is discused [3]. The closed time-like curve can be constructed by means of attaching of four-dimensional handle (4-wormhole).

When we have the chance to create the 4-wormhole going from the Present to the Past? It is evivently if the temporal stream carries out the peace D_0 to the Past which is lying arbitrarily nearly. We can realize this by means of the theory of resilient leaves of foliations of codimension 1 in the 5-dimensional Lorentz manifold and the conformal Kaluza-Klein theory.

Let $< V^4, g_{ik} >$ be a leaf of an orientable foliation $\mathcal{F}$ of codimension 1 in the 5-dimensional Lorentz manifold $< V^5, G_{AB} >$, $g = G\,|_{V^4}$, $A, B = 0, 1, 2, 3, 5$. Foliation $\mathcal{F}$ is determinated by the differential 1-form $\gamma = \gamma_A dx^A$. If the Godbillon-Vey class $GV(\mathcal{F}) \neq 0$ then the foliation $\mathcal{F}$ has a resilient leaves.

We suppose that real global space-time V^4 is a resilient one, i.e. is a resilient leaf of some foliation $\mathcal{F}$. Hence there exists an arbitrarily small neighborhood $U_a \subset V^5$ of the event $a \in V^4$ such that $U_a \cap V^4$ consists of at least two connected components U_a^1 and U_a^2.

Remove the 4-dimensional balls $B_a \subset U_a^1, B_b \subset U_a^2$, where an event $b \in U_a^2$, and join the boundaries of formed two holes by means of 4-dimensional cylinder. As result we have a 4-wormhole C, which is a Time machine if b belongs to the past of event a. The past of a is lying arbitrarily nearly. The distant Past is more accessible than the near Past. A movement along 5-th coordinate (in the direction γ^A) gives the infinite piercing of space-time V^4 at the points of Past and Future. It is the property of a resilient leaf.

Define the Lorentz metric (temporal stream) $\tilde{G}_{AB}$ on V^5 in the following way

$$\tilde{G}_{AB} = -\gamma_A \gamma_B + \tilde{g}_{AB}, \tag{1}$$

$$\tilde{g}_{5A} = 0, \tag{2}$$

where $\tilde{g}_{AB}$ is metric tensor of V^4. It is more convenient [4] to use conformal metric G_{AB}

$$G_{AB} = \phi^{-2}\tilde{G}_{AB}, \qquad g_{AB} = \phi^{-2}\tilde{g}_{AB}, \qquad \phi = \gamma_5, \tag{3}$$

$$G_{AB} = -\lambda_A \lambda_B + g_{AB}, \tag{4}$$

$$\lambda = \phi^{-1}\gamma, \tag{5}$$

$$(d\tilde{I})^2 = \tilde{G}_{AB} dx^A dx^B = \phi^2 G_{AB} dx^A dx^B = \phi^2 dI^2 \tag{6}$$

with the cylindrical condition that G_{AB} are not depend of x^5 and $G_{55} = -1$. Than ϕ is a scalar field, and the 5-dimensional Einstein's equations

$$R^{(5)}_{AB} - \frac{1}{2} G_{AB} R^{(5)} = \kappa Q_{AB} \tag{7}$$

are reduced [4] to the 4-dimensional Einstein's equations, the Maxwell's equations and the Klein-Fock equation for ϕ.

If σ is the characteristic 2-dimensional section of the 3-dimensional domain D that contains the time machine than the mean value of energy density jump under the start of machine [3]

$$< \delta\epsilon > \sim \frac{c^4}{4\pi G} \frac{1}{\sigma},$$

where c is the light velocity, G is the gravitational constant.

The creation of 4-wormhole means that 3-dimensional piece D_0 leaves the 3-dimensional physical space V^3 or is separated from V^3.

Define the proper time along time-like curve L as

$$d\tau = \frac{dI}{c}, \qquad dI^2 = G_{AB} dx^A dx^B, \tag{8}$$

where coordinates $x^A (A = 0, 1, 2, 3, 5)$ are given in domain $U_a \subset V^5$. Suppose that Time machine moves in the Past along L in U_a such that it is at rest in the domain $V^4 \cap U_a$, i.e. $x^1, x^2, x^3 = const$. Then

$$dI^2 = ds^2 - d\lambda^2, \qquad ds^2 = c^2 dt^2 - dl^2, \tag{9}$$

where

$$dt = \frac{g_{0i} dx^i}{c\sqrt{g_{00}}}, \qquad dl^2 = \left(-g_{\alpha\beta} + \frac{g_{0\alpha} g_{0\beta}}{g_{00}} \right) dx^\alpha dx^\beta \qquad (\alpha, \beta = 1, 2, 3) \tag{10}$$

are chronometrical invariants respectively time and length in space-time V^4. Hence

$$d\tau = \sqrt{1 - \left(\frac{d\lambda}{ds}\right)^2} \frac{ds}{c} = \sqrt{1 - \left(\frac{d\lambda}{ds}\right)^2} \sqrt{1 - \left(\frac{dl}{cdt}\right)^2} dt = \sqrt{1 - \left(\frac{d\lambda}{ds}\right)^2} dt, \tag{11}$$

since $dl = 0$.

Let L be a time-like geodesic curve with respect to 5-metric G_{AB} that has the ends a and b.

Then [4]

$$\frac{d\lambda}{ds} = -\frac{e}{2m\sqrt{G}}, \tag{12}$$

where e is electric charge, and m is mass of Time machine.

If any vector ξ is tangent to V^4 then $d\lambda(\xi) = \lambda_A \xi^A = 0$. Hence the motion along curve $L : x^A = x^A(s)$ and which is transversal to W^4 is characterized by means of the inequality

$$d\lambda(\frac{dx^A}{ds}) = \lambda_A \frac{dx^A}{ds} = \frac{d\lambda}{ds} \neq 0. \tag{13}$$

The relations (12) and (13) imply that for transversal motion, i.e. motion in 5-th dimension it is nesessary that body had an electric charge. Therefore for start of Time machine we must give to it the electric charge.

It follows from (11) that $(d\lambda/ds)^2 \leq 1$, because time τ must be real. Hence $e/2m\sqrt{G} \leq 1$. This inequality is not correct for electron. Thus there exists the restriction for mass and electric charge of Time machine.

In the case when the Godbillon-Vey class $GV(\mathcal{F}) = 0$ one can attempt to change this, i.e. to include the foliation $\mathcal{F}$ in smooth one-parametric family of foliations $\mathcal{F}_\mu$ characteristic class

$$Char_{\mathcal{F}_\mu}(\alpha) = GV(\mathcal{F}_\mu), \alpha \in H^3(W_1), \tag{14}$$

$$Char_{\mathcal{F}_\mu}(\alpha) : H^*(W_1) \to H^*(V^5, \mathbb{R}) \tag{15}$$

of which is changed with change of parameter μ in accord to law

$$GV(\mathcal{F}_\mu)\frac{d}{d\mu}GV(\mathcal{F}_\mu) = 0. \tag{16}$$

Change (variation) is possible if $\alpha \in H^3(W_1)$ does not belong to image of homomorphism of inclusion $H^3(W_2) \to H^3(W_1)$. Since $H^3(W_2) = 0$ then one can be taken abitrary cohomological class $\alpha \neq 0$.

Suppose that our space-time is not resilient one. Can it be rolled up? It is very difficult question. As we think it has positive answer. The energy which one is need for such "rolling up" can be estimated by means of formulas of theory foliations.

References

1. M.S. Morris, K.S. Thorne, U. Yurtsever, *Phys. Rev. Lett.* **61**, 1446 (1988).
2. M. Konstantinov, *Izvestiya vuzov. Fizika.* **12**, 84 (1992) (Russian).
3. A.K. Guts, gr-qc/9612064.
4. Yu.S. Vladimirov, *"Dimension of physical space-time and the union of interections".* – Moscow, Moscow State University, 1987. (Russian).