INTERACTION OF A PLANAR GRAVITATIONAL WAVE WITH A SCALAR FIELD

A. K. Guts

UDC 530.12:531.51

It is shown that a strong gravitational wave can reflect a flow of scalar particles moving toward it. The particles initially penetrate the forward wave propagation front and only then are reflected.

We shall consider the fate of a scalar field which propagates toward a planar gravitational wave. In [1] it was shown that a planar electromagnetic wave colliding with a gravitational wave packet first penetrates beyond the forward fron of packet propagation, and then reverses its propagation direction, i.e., is reflected. Below we will demonstrate that a similar phenomenon occurs upon collision of a Peres gravitational wave with a propagating scalar field. The Peres metric satisfied the various criteria of gravitational radiation [2]. Therefore, the result obtained is valid no matter of the view taken as to the nature of gravitational waves. The Peres metric is of further interest because in a number of cases it may prove to be a source generating such waves [3].

We will assume that a planar gravitational wave propagates in the positive x-direction, while the scalar field moves in the opposite direction. The gravitational wave propagation front is specified by the expression $x^0 = ct - x = \mathrm{const}$, while that of the scalar wave-particle is given by $x^1 = 1/\alpha[ct + (\alpha - 1)x] = \mathrm{const}$. Here $\alpha \geqslant 2$ is a constant related to the particle speed v as follows: $v = c/(\alpha - 1)$. The Minkowsky metric $ds^2 = c^2dt^2 - dx^2 - dz^2 - dz^2$ in coordinates $x^0 = ct - x$, $x^1 = 1/\alpha[ct + (\alpha - 1)x]$, $x^2 = y$, $x^3 = z$ has the form:

$$ds^2 = (1 + 2\beta)\, dx^{0^2} + 2dx^0dx^1 - dx^{2^2} - dx^{3^2}, \quad \beta = -1/\alpha. \tag{1}$$

We will assume that the portion of space–time which corresponds to particle motion up to the moment of collision is specified by the inequalities $x^0 \leqslant 0$, $x^1 \geqslant 0$, while the portion of space–time corresponding to wave propagation until the collision moment is given by $x^0 \geqslant 0$, $x^1 \leqslant 0$. The collision occurs at the moment $x^0 = 0$ at $x^1 = 0$. The forward propagation front of the Peres wave is described by the equation $x^0 = 0$. The flow of time from past to future corresponds to increase in the coordinate x^0.

We will neglect the contribution of the scalar field to space–time curvature and at $x^0 < 0$ we will take the metric in the form of Eq. (1), while for $x^0 \geqslant 0$ we use the Peres metric:

$$ds^2 = (1 + 2\beta + f(x^0,\ x^2,\ x^3))\, dx^{0^2} + 2dx^0dx^1 - dx^{2^2} - dx^{3^2}, \tag{2}$$

where

$$\frac{\partial^2 f}{\partial x^{2^2}} + \frac{\partial^2 f}{\partial x^{3^2}} = 0 \tag{3}$$

(see [2, p. 113]). Satisfaction of Eq. (3) is equivalent to the assumption that metric (2) satisfied the Einstein equations for a vacuum, $R_{ik} = 0$. The function f must not only satisfy Eq. (3), but must also be chosen so that the curvature of space–time is nonzero. For this purpose, it is useful to consider that

---

Omsk State University. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 1, pp. 46-49, January, 1983. Original article submitted October 8, 1980.

 0038-5697/83/2601-0046$07.50

$$R_{2020} = -\frac{1}{2}\frac{\partial^2 f}{\partial x^{2^2}}, \quad R_{2030} = R_{2023} = -\frac{1}{2}\frac{\partial^2 f}{\partial x^2 \partial x^3}, \quad R_{3030} = -\frac{1}{2}\frac{\partial^2 f}{\partial x^{3^2}},$$

where R_{iklm} are the components of the Riemann-Kristoffel tensor. We assume that

$$f(0, x^2, x^3) = 0. \tag{3'}$$

This condition assures a continuous maerger of the metrics of Eqs. (1) and (2).

In a pseudo-Riemannian manifold the equation of the scalar field has the following form:

$$\sqrt{-g}\mu^2\Psi + \frac{\partial}{\partial x^\kappa}\left(\sqrt{-g}\, g^{i\kappa}\frac{\partial\Psi}{\partial x^i}\right) = 0 \tag{4}$$

or

$$\mu^2\Psi + 2\frac{\partial^2\Psi}{\partial x^0 \partial x^1} - (1 + 2\beta + f)\frac{\partial^2\Psi}{\partial x^{1^2}} - \frac{\partial^2\Psi}{\partial x^{2^2}} - \frac{\partial^2\Psi}{\partial x^{3^2}} = 0, \tag{4'}$$

where $\mu = mc/\hbar$ [4, p. 127]. We will calculate the density of the particle three-momentum $P_{(\alpha)}$ (polarized energy flow through a unit area per unit time) [5, p. 146] with respect to the following orthoreference:

$$\begin{aligned}
\lambda^i_{(0)} &= \left(1,\ -\frac{1}{2}(f + 2\beta),\ 0,\ 0\right); \\
\lambda^i_{(1)} &= \left(-1,\ \frac{1}{2}(f + 2 + 2\beta),\ 0,\ 0\right), \\
\lambda^i_{(2)} &= (0,\ 0,\ 1,\ 0); \\
\lambda^i_{(3)} &= (0,\ 0,\ 0,\ 1);
\end{aligned} \tag{5}$$

where $g_{ik}\lambda^i_{(m)}\lambda^k_{(n)} = \eta_{mn}$ and $\eta_{mn} = \mathrm{diag}\{1, -1, -1, -1\}$ is the Minkowsky tensor. Then $P_{(\alpha)} = -T_{i\kappa}\lambda^i_{(0)}\lambda^\kappa_{(\alpha)}$, $(\alpha = 1, 2, 3)$, where

$$T_{i\kappa} = \frac{\partial\Psi}{\partial x^i}\frac{\partial\Psi}{\partial x^\kappa} + \frac{1}{2}g_{i\kappa}\left(\mu^2\Psi^2 - g^{mn}\frac{\partial\Psi}{\partial x^m}\frac{\partial\Psi}{\partial x^n}\right) -$$

is the scalar field energy-momentum tensor.

Performing the calculations we obtain:

$$P_{(1)} = \left[\frac{1}{2}\Psi_1\cdot(1 + 2\beta + f) - \Psi_0\right]^2 - \frac{1}{4}\Psi_1^2 = -\left[\Psi_0 - \frac{1}{2}(f + 2\beta)\cdot\Psi_1\right]\left[\frac{1}{2}(2 + 2\beta + f)\Psi_1 - \Psi_0\right];$$

$$\begin{aligned}
P_{(2)} &= -\left[\Psi_0 - \frac{1}{2}(f + 2\beta)\cdot\Psi_1\right]\cdot\Psi_2; \\
P_{(3)} &= -\left[\Psi_0 - \frac{1}{2}(f + 2\beta)\cdot\Psi_1\right]\cdot\Psi_3,
\end{aligned}$$

where $\Psi_i \equiv \partial\Psi/\partial x^i$.

We assume that all functions Ψ_i are finite in the variable x^0; Ψ_2, Ψ_3 maintain their sign for any x^0, and finally, $\Psi_1 \neq 0$ ($\Psi_1 \neq 0$, since we are considering a particle moving toward the gravitational wave). These limitations are not burdensome, and in general correspond to the real physical problem (see example below). Before the collision $\Psi = \Psi(x^1, x^2, x^3)$, i.e., is independent of x^0, $f \equiv 0$, therefore $P_{(1)} = \beta(1 + \beta)\Psi_1^2 < 0$. If we now consider $P_{(\alpha)}$ ($\alpha = 1$, 2, 3) as functions of the variable f, it can easily be seen that they all change sign simultaneously at one and the same value of $f = f_0$ as $|f|$ changes from zero to infinity. But $f = f(x^0, x^2, x^3)$. Therefore, we may assume that the change in f occurs upon increase of the variable x^0 from zero to infinity. Let $f_0 = f(x^0_0, x^2, x^3)$, where $x^0_0 \geqslant 0$. Thus, the scalar particles which have traversed the forward propagation front of the gravitational wave cannot penetrate beyond the front $x^0 = x^0_0$, at which a radical change occurs in the direction of particle flux propagation. In essence reflection of the particle flux by the strong gravitational wave occurs. We note that our conclusions are identically valid for head-on or oblique "collision" of the fields.

Examples. 1. We will consider a massless particle with $\Psi \equiv x^1$, satisfying the conditions enumerated above. In this case ($\mu = 0$, $\beta = -^1/_2$)

$$P_{(\alpha)} = \left(\frac{1}{4} f^2 - \frac{1}{4},\ 0,\ 0\right)$$

and reflection occurs when there is a $x^0_0 > 0$ such that $|f(x^0_0, x^2, x^3)| > 1$.

The question then arises of whether such a function f exists. Indeed that function must satisfy Eqs. (3), (3') and depend significantly on x^2, x^3, since the latter ensure curvature of space–time. It can easily be seen that the desired function can have the form

$$f(x^0, x^2, x^3) = x^{0^2} \cdot [1 + \sqrt{x^2 + \sqrt{x^{2^2} + x^{3^2}}}],\ x^0_0 = 2.$$

On the other hand, if we take

$$f(x^0, x^2, x^3) = \frac{1}{\pi} (\sin x^0) \cdot \operatorname{arctg} \frac{x^3}{x^2},\ \text{то}\ |f(x^0, x^2, x^3)| < 1 \tag{6}$$

reflection does not take place. This result is valid, because inequality (6) indicates the relative weakness of the gravitational wave. If we take the following functions: $f = x^0 x^2 x^3$ or $f = x^0 \ln(x^2 + x^3)$, then in the plane $x^2 x^3$ orthogonal to the direction of particle flux motion "gaps" appear, through which the particles pass through the gravitational wave packet. In the first case the "gap" is described by the equation $x^2 x^3 = 0$, and in the second, by $x^2 + x^3 = 1$.

2. We will consider massive particle. The solution of Eq. (4') will be a function $\Psi(x^1, x^2, x^3) = x^1 \varphi(x^2, x^3)$, where $\varphi(x^2, x^3) \neq 0$ satisfies the equation $\Delta \varphi = \mu^2 \varphi$. We see that Ψ_1 is finite in x^0, $\Psi_1 = \varphi \neq 0$. Consequently, for the solution considered the reflection phenomenon will be observed.

Thus, scalar particles colliding with a strong Peres gravitational wave must change the direction of their propagation, and moreover, the phenomenon which we have termed reflection will occur. This phenomenon may be described by the statement that the particles do not penetrate beyond some well defined gravitational wave propagation front. But in no case does reflection occur from the forward wave propagation front.

LITERATURE CITED

1. Yu. G. Sbytov, Zh. Eksp. Teor. Fiz., 63, 737 (1972).
2. V. D. Zakharov, Gravitational Waves and Einstein's Theory of Gravitation [in Russian], Nauka, Moscow (1972).
3. Yu. P. Vlasov, "Application of canonical Arnovit–Dezer–Mizner formalism to exact wave solutions," Preprint Inst. Teor. Fiz. Akad. Nauk SSSR, ITF-71-48R, Kiev (1971).
4. N. V. Mitskevich, Physical Fields in the General Theory of Relativity [in Russian], Nauka, Moscow (1969).
5. J. Sing, General Theory of Relativity [Russian translation], IL, Moscow (1963).