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THE DEUTSCH THEORY OF THE MULTIVERSE AND PHYSICAL CONSTANTS[1]

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The Deutsch multiverse is a collection of parallel universes. In this article, a formal theory and a topos-theoretic model of the Deutsch multiverse are given. For this purpose, the Lawvere-Kock Synthetic Differential Geometry and topos models for smooth infinitesimal analysis are used. Physical properties of multi-variant and many-dimensional parallel universes are discussed. The source of multiplicity of physical objects is the set of physical constants.

Теория мультиверса Дойча и физические константы
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Мультиверс Дойча — это множество параллельных миров, описываемое в рамках эвереттовской интерпретации квантовой теории. В статье предлагается формальная теория мультиверса на основе синтетической дифференциальной геометрии Ловера-Кока и рассматриваются ее гладкие теоретико-топосные модели. Физические объекты становятся многовариантными, каждый вариант реализуется в параллельной гипервселенной. Источником многовариантности являются физические константы.

1. Intoduction

Deutsch's book [1] gives a sketch of the structure of physical reality named the Multiverse, which is a set of parallel universes. A correct description of the Multiverse can be made only in the framework of quantum theory.

In this article, a formal theory and a topos-theoretic model of the Deutsch multiverse are given.

We wish to preserve the framework of mathematical tools of 4-dimensional General Relativity, and so we shall consider the Universe as a concrete 4-dimensional Lorentzian manifold $\langle \mathcal{R}^4, g^{(4)} \rangle$ (named space-time).

2. Formal theory of the Multiverse

We construct our theory of the Multiverse as a formal theory $\mathcal{T}$ which is maximally similar to General Relativity, i.e., as a theory of *one* 4-dimensional Universe, but other parallel universes must appear under costruction of models of the formal theory.

The basis of our formal theory $\mathcal{T}$ is the Kock-Lawvere Synthetic Differential Geometry (SDG) [2].

It is important to say that SDG has no set-theoretic model because the Lawvere-Kock axiom is incompatible with the Law of excluded middle. Hence we shall construct a formal theory of the Multiverse on the basis of intuitionistic logic. Models for this theory are smooth topos-theoretic models, and for their description the usual classical logic is used.

In SDG, the commutative ring $\mathcal{R}$ is used instead of the real field $\mathbb{R}$. The ring $\mathcal{R}$ must satisfy the following

Lawvere-Kock axiom. *Let* $D = \{x \in \mathcal{R} : x^2 = 0\}$. *Then*

$$\forall(f \in \mathcal{R}^D)\exists!(a, b) \in \mathcal{R} \times \mathcal{R}\ \forall d \in D(f(d) = a + b \cdot d).$$

and some other axioms (see in [3, Ch.VII].).

The ring $\mathcal{R}$ includes real numbers from $\mathbb{R}$ and has new elements named *infinitesimals* belonging to the "sets"

$$D = \{d \in \mathcal{R} : d^2 = 0\}, ..., D_k = \{d \in \mathcal{R} : d^{k+1} = 0\}, ...$$
$$\Delta = \{x \in \mathcal{R} : f(x) = 0,\ \text{all}\ f \in m^g_{\{0\}}\},$$

where $m^g_{\{0\}}$ is the ideal of smooth functions having a zero germ at 0, i.e., vanishing in a neighbourhood of 0.

We have

$$D \subset D_2 \subset ... \subset D_k \subset ... \subset \Delta.$$

We can construct a Riemannian geometry for the four-dimensional (formal) manifolds $\langle \mathcal{R}^4, g^{(4)} \rangle$. These manifolds are the basis for the Einstein theory of gravitation [4].

We postulate that *the Multiverse is four-dimensional space-time in SDG, i.e., a formal Lorentzian manifold* $\langle \mathcal{R}^4, g^{(4)} \rangle$ *for which the Einstein field equations hold:*

$$R^{(4)}_{ik} - \frac{1}{2}g^{(4)}_{ik}(R^{(4)} - 2\Lambda) = \frac{8\pi G}{c^4}T_{ik}. \quad (1)$$

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A solution of these equations is the 4-metric $g^{(4)}(x)$, $x \in \mathcal{R}$.

Below we consider the physical consequences of our theory in the so-called well-adapted smooth topos models of the form $\mathbf{Set}^{\mathbb{L}^{op}}$ which contain, as a full subcategory, the category of smooth manifolds $\mathcal{M}$.

3. Smooth topos models of the Multiverse

Let $\mathbb{L}$ be the dual category for the category of finitely generated C^∞-rings. It is called a *category of loci* [3]. Objects of $\mathbb{L}$ are finitely generated C^∞-rings, and morphisms are reversed morphisms of the category of finitely generated C^∞-rings.

The object (locus) of $\mathbb{L}$ is denoted as ℓA, where A is a C^∞-ring. Hence, the $\mathbb{L}$-morphism $\ell A \to \ell B$ is the C^∞-homomorphism $B \to A$.

A finitely generated C^∞-ring ℓA is isomorphic to a ring of the form $C^\infty(\mathbb{R}^n)/I$ (for some natural number n and some ideal I of finitely generated functions).

The category $\mathbf{Set}^{\mathbb{L}^{op}}$ is a topos [3]. We consider the topos $\mathbf{Set}^{\mathbb{L}^{op}}$ as a model of a formal theory of the Multiverse.

From Deutsch's point of view, the transition to a concrete model of a formal theory is creation of *virtual reality*[3]. Physical Reality that we perceive was called by Deutsch *the Multiverse*[4]. Physical Reality is also virtual reality which was created by our brain [1, p.140].

A model of the Multiverse is a *generator of virtual reality* which has some *repertoire of environments.* A generator of virtual reality creates environments, and we observe them. Let us explain it.

Under the interpretation $i : \mathbf{Set}^{\mathbb{L}^{op}} \models \mathcal{T}$ of the formal Multiverse theory $\mathcal{T}$ in the topos $\mathbf{Set}^{\mathbb{L}^{\mathbf{op}}}$, objects of the theory, for example, the ring $\mathcal{R}$, the power $\mathcal{R}^{\mathcal{R}}$ and so on are interpreted as objects of the topos, i.e., functors $F = i(\mathcal{R})$, $F^F = i(\mathcal{R}^{\mathcal{R}})$ and so on. Maps, e.g., $\mathcal{R} \to \mathcal{R}$, $\mathcal{R} \to \mathcal{R}^{\mathcal{R}}$ are now morphisms of the topos $\mathbf{Set}^{\mathbb{L}^{op}}$, i.e., natural transformations of functors: $F \to F$, $F \to F^F$.

Finally, under an interpretation of the language of a formal Multiverse theory, we must interpret elements of the ring $\mathcal{R}$ as "elements" of the functors $F \in \mathbf{Set}^{\mathbb{L}^{op}}$. In other words, we must give an interpretation of the relation $r \in \mathcal{R}$. It is a very difficult problem because the functor F is defined on the category of loci $\mathbb{L}$; its independent variable is an arbitrary locus ℓA, and the dependent variable is a set $F(\ell A) \in \mathbf{Set}$. To solve this problem, we consider *generalized elements* $x \in_{\ell A} F$ of the functor F.

A generalized element $x \in_{\ell A} F$, or an *element* x *of the functor* F *at stage* ℓA, is called an element $x \in F(\ell A)$.

Now we interpret the element $r \in \mathcal{R}$ as a generalized element $i(r) \in_{\ell A} F$, where $F = i(\mathcal{R})$. We have as many such elements as many loci. A transition to the model $\mathbf{Set}^{\mathbb{L}^{op}}$ causes "reproduction" of the element r. It begins to exist in an infinite number of variants $\{i(r) : i(r) \in_{\ell A} F, \ell A \in \mathbb{L}\}$.

Note that since the 4-metric $g^{(4)}$ is an element of the object $\mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}$, the "intuitionistic" 4-metric begins to exist in an infinite number of variants $i(g)^{(4)} \in_{\ell A} i(\mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4})$. Denote such a variant as $i(g)^{(4)}(\ell A)$.

To simplify the interpretation, we shall operate with objects of models $\mathbf{Set}^{\mathbb{L}^{op}}$. In other words, we shall write $g^{(4)}(\ell A)$ instead of $i(g)^{(4)}(\ell A)$.

Every variant $g^{(4)}(\ell A)$ of the 4-metric $g^{(4)}$ satisfies its "own" Einstein equations [4]

$$R^{(4)}_{ik}(\ell A) - \frac{1}{2} g^{(4)}_{ik}(\ell A)[R^{(4)}(\ell A) - 2\Lambda(\ell A)] = \frac{8\pi G}{c^4} T_{ik}(\ell A).$$

(The constants c and G can also have different values at different stages ℓA).

It follows from the theory that, when $\ell A = \ell C^\infty(\mathbb{R}^m)$, then

$$g^{(4)}(\ell A) = [g \in_{\ell A} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}] \equiv g^{(4)}_{ik}(x^0, ..., x^3, a) dx^i dx^k, \quad a = (a^1, ..., a^m) \in \mathbb{R}^m.$$

We extend the four-dimensional metric $g^{(4)}_{ik}(x^0, ..., x^3, a)$ to a $(4+m)$-metric in the space $\mathbb{R}^{4+m}$:

$$g^{(4+m)}_{AB} dx^A dx^B \equiv \equiv g^{(4)}_{ik}(x^0, ..., x^3, a) dx^i dx^k - da^{1^2} - ... - da^{m^2}. \quad (2)$$

We get the $(4+m)$-dimensional pseudo-Riemannian geometry $\langle \mathbb{R}^{4+m}, g^{(4+m)}_{AB} \rangle$.

Symbolically, the procedure of creation of multidimensional variants of the space-time geometry by means of the intuitionistic 4-geometry $\langle \mathcal{R}^4, g^{(4)} \rangle$ can be represented as the formal sum

$$g^{(4)} = c_0 \cdot \underbrace{[g^{(4)} \in_{\mathbf{1}} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}]}_{\text{4-geometry}} + c_1 \cdot \underbrace{[g^{(4)} \in_{\ell C^\infty(\mathbb{R}^1)} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}]}_{\text{5-geometry}} + ... \\ ... + c_{n-4} \cdot \underbrace{[g^{(4)} \in_{\ell C^\infty(\mathbb{R}^{n-4})} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}]}_{\text{n-geometry}} + ...,$$

where the coefficients c_m are taken from the field of complex numbers.

Since the number of stages is infinite, we must write an integral instead of the sum:

$$g^{(4)} = \int_{\mathbb{L}} \mathcal{D}[\ell A] c(\ell A) [g^{(4)} \in_{\ell C^\infty(\mathbb{R}^{n-4})} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}]. \quad (3)$$

[3] This thought belongs to Artem Zvyagintsev.

[4] Multiverse = many (multi-) worlds; a universe is one (uni-) world.

Using the notations of quantum mechanics[5],

$$g^{(4)} \to |g^{(4)}\rangle,\ [g^{(4)} \in_{\ell C^\infty(\mathbb{R}^{n-4})} \mathcal{R}^{\mathcal{R}^4\times\mathcal{R}^4}] \to |g^{(4)}(\ell A)\rangle.$$

Then (3) is rewritten in the form

$$|g^{(4)}\rangle = \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] c(\ell A) |g^{(4)}(\ell A)\rangle. \tag{4}$$

Consequently, formally the Lawvere-Kock 4-geometry $\langle\, \mathcal{R}^4, g^{(4)}\,\rangle$ is the infinite sum

$$\mathcal{R}^4 = \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] c(\ell A) \mathcal{R}^4_{\ell A}$$

of classical multidimensional pseudo-Riemmanian geometries $\mathcal{R}^4_{\ell A} = \langle\, \mathbb{R}^{4+m}, g^{(4+m)}_{AB}(x,a)\,\rangle$ any of which contains a foliation of 4-dimensional parallel universes (leaves) (under fixed $a = \text{const}$). The geometric properties of these universes, as was shown in [6, 7], are different even within the same stage ℓA.

Now we recall the environments of virtual reality which must appear when referring to a Multiverse model, in this instance, to the model $\mathbf{Set}^{\mathbb{L}^{op}}$. This model is a generator of virtual reality. It is not difficult to understand that the generalised element $|g^{(4)}(\ell A)\rangle$ is a metric of a specific environment (=hyperspace $\mathcal{R}^4_{\ell A}$) with the "number" ℓA. In other words, the study of any object of the theory $\mathcal{T}$ at stage ℓA is a transition to one of the environments from the repertoire of the virtual reality generator $\mathbf{Set}^{\mathbb{L}^{op}}$.

4. The Gödel-Deutsch Multiverse

As an example of the Multiverse, consider the cosmological solution obtained by Kurt Gödel [8]:

$$g^{(4)}_{ik} = \alpha^2 \begin{pmatrix} 1 & 0 & e^{x^1} & 0 \\ 0 & -1 & 0 & 0 \\ e^{x^1} & 0 & e^{2x^1}/2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{5}$$

This metric satisfies the Einstein equations (1) with the energy-momentum tensor of dust matter

$$T_{ik} = c^2 \rho u_i u_k,$$

if

$$\frac{1}{\alpha^2} = \frac{8\pi G}{c^2}\rho, \qquad \Lambda = -\frac{1}{2\alpha^2} = -\frac{4\pi G\rho}{c^2}. \tag{6}$$

Take

$$\alpha = \alpha_0 + d, \qquad \Lambda = \Lambda_0 + \lambda, \qquad \rho = \rho_0 + \varrho, \tag{7}$$

where $d,\ \lambda,\ \varrho \in D$ are infinitesimals, and substitute these into (6). We get

$$\begin{aligned} \frac{1}{(\alpha_0+d)^2} &= \frac{1}{\alpha_0^2} - \frac{2d}{\alpha_0^3} = \frac{8\pi G}{c^2}(\rho_0 + \varrho), \\ 2\Lambda_0 + 2\lambda &= -\frac{1}{\alpha_0^2} + \frac{2d}{\alpha_0^3}, \\ \Lambda_0 + \lambda &= -\frac{4\pi G\rho_0}{c^2} - \frac{4\pi G\varrho}{c^2}. \end{aligned}$$

Suppose that $\alpha_0,\ \Lambda_0,\ \rho_0 \in \mathbb{R}$ satisfy the relations (6). Then

$$\lambda = -\frac{4\pi G}{c^2}\varrho, \qquad d = -\frac{4\pi G\alpha_0^3}{c^2}\varrho.$$

Under the interpretation in the smooth topos $\mathbf{Set}^{\mathbb{L}^{op}}$, the infinitesimal $\varrho \in D$ at the stage $\ell A = C^\infty(\mathbb{R}^m)/I$ is a class of smooth functions of the form $\varrho(a)\ mod\ I$, where $[\varrho(a)]^2 \in I$ [3, p.77].

Consider the properties of the Gödel-Deutsch multiverse at the stage $\ell A = \ell C^\infty(\mathbb{R})/(a^4)$ [6], where $a \in \mathbb{R}$. Obviously, it is possible to take an infinitesimal of the form $\varrho(a) = a^2$. The Multiverse at this stage is a 5-dimensional hyperspace. This hyperspace contains a foliation whose leaves are defined by the equation $a = \text{const}$. The leaves are parallel universes in the hyperspace (environment) $\mathcal{R}^4_{\ell A}$ with the metric $g^{(4)}(\ell A) = g^{(4)}_{ik}(x,a)$ defined by Eqs. (5), (7). The density of dust matter $\rho = \rho_0 + \varrho(a)$ grows from the classical value $\rho_0 \sim 2\cdot 10^{-31}$ g/cm^3 to $+\infty$ as $a \to \pm\infty$. The cosmological constant also grows infinitely to $-\infty$. Hence the parallel universes have physical properties different from those of our Universe.

At the stage $\ell A = \ell C^\infty(\mathbb{R})/(a^2)$, $\varrho(a) = a$ and $\rho = \rho_0 + \varrho(a) \to -\infty$ as $a \to -\infty$, i.e., ρ is not physically interpreted (we have "exotic" matter with negative density).

Finally, at the stage $\mathbf{1} = \ell C^\infty(\mathbb{R})/(a)$ all $\varrho(a) = d(a) = \lambda(a) = 0$, i.e., we have the classical Gödel universe.

5. The Friedmann-Deutsch Multiverse

Now we consider Friedmann's closed model of the Universe, which, in the coordinates $(x^0, \chi, \theta, \varphi)$, $x^0 = ct$, has the following metric:

$$\begin{aligned} ds^2 &= g^{(4)}_{ik} dx^i dx^k \\ &= c^2dt^2 - R^2(t)[d\chi^2 + \sin^2\chi\ (d\theta^2 + \sin^2\theta d\varphi^2)]. \end{aligned} \tag{8}$$

This metric satisfies the Einstein equations with the energy-momentum tensor of dust matter

$$T_{ik} = c^2 \rho u_i u_k,$$

[5]Dirac's notations are: $|P\rangle = \psi(\xi)\rangle \equiv \psi(\xi)$; in our case $\psi(\xi)$ is $g^{(4)}$ (a representative of the state $|P\rangle$), and $|P\rangle$ is $|g^{(4)}\rangle$ [5, p.111-112].

[6]Here $(f_1, ..., f_k)$ is an ideal of the ring $C^\infty(\mathbb{R}^n)$ generated by the functions $f_1, ..., f_k \in C^\infty(\mathbb{R}^n)$, i.e., having the form $\sum_{i=1}^k g_i f_i$, where $g_1, ..., g_k \in C^\infty(\mathbb{R}^n)$ are arbitrary smooth functions.

under the condition that

$$\rho R^3(t) = \text{const} = \frac{M}{2\pi^2}, \tag{9}$$

$$R = R_0(1-\cos\eta), \qquad t = \frac{R_0}{c}(\zeta - \sin\eta), \tag{10}$$

$$R_0 = \frac{2GM}{3\pi c^3}, \tag{11}$$

where M is sum of body masses in 3-space [9, p.438].

Let

$$G = k + d, \qquad d \in D, \tag{12}$$

where $k = 6,67 \cdot 10^{-8}$ $[CGS]$ is the classical gravitational constant.

At the stage $\mathbf{1} = \ell C^\infty(\mathbb{R})/(a)$, $d(a) = 0$, i.e., we have the classical Friedmann Universe.

Consider the state of the Friedmann-Deutsch multiverse at the stage $\ell A = \ell C^\infty(\mathbb{R})/(a^4)$, where $a \in \mathbb{R}$. It is obviously possible to take an infinitesimal of the form $d(a) = a^2$. The Multiverse at this stage is a 5-dimensional hyperspace. This hyperspace contains a foliation whose leaves are defined by the equation $a = \text{const}$. The leaves are parallel universes in the hyperspace (environment) $R^4_{\ell A}$ with the metric $g^{(4)}(\ell A) = g^{(4)}_{ik}(x,a)$ defined by Eqs. (8)–(11).

The radius of a "Universe" with the number $a = \text{const}$ and the dust density following from (9) is equal to

$$\begin{aligned} R &= \frac{2M}{3\pi c^3}(k + a^2)(1-\cos\eta), \\ \rho(a) &= \frac{27\pi c^9}{16k^3M^2(1-\cos\eta)^3}\left(1 - \frac{3}{k}d(a)\right). \end{aligned}$$

So, with $d = a^2$, the radius of parallel universes with numbers $|a| \to +\infty$ grows to $+\infty$. The dust density $\rho(a)$ decreases, then $\rho(a)$ crosses zero and becomes negative, $\rho(a) \to -\infty$ as $|a| \to +\infty$. All this tells us that the parallel universes can have physical characteristics which are absolutely different from the characteristics of our Universe.

6. Transitions between parallel hyperspaces

A change of the stage ℓA into the stage ℓB is a morphism between the two stages,

$$\ell B \xrightarrow{\Phi} \ell A.$$

When $\ell A = \ell C^\infty(\mathbb{R}^n)$ and $\ell B = \ell C^\infty(\mathbb{R}^m)$, the transition Φ between the stages gives the smooth mapping

$$\phi : \mathbb{R}^m \ni b \to a \in \mathbb{R}^n, \qquad a = \phi(b).$$

Hence if the constants are $G = G(a)$ and $\Lambda = \Lambda(a)$ at the stage ℓA, then we have at the new stage ℓB: $G = G(\phi(b))$ and $\Lambda = \Lambda(\phi(b))$. In other words, the dependence of the physical constants on the extra dimensions is transformed into their dependence on some extra field ϕ. This fact can be useful in connection with the investigations concerning the introduction of an effective gravitational constant depending on some scalar field (see, e.g., [10]).

7. Conclusion

As follows from Sections 4 and 5, a source of multiplicity of objects and the appearance of parallel hyperspaces are the physical constants (such as ρ, Λ, G). A reason for this is the following. Traditionally we consider the physical constants as real numbers. It means an impossibility of findings their exact values. So we must assume that a physical constant is $K = K_0 + d$ where d is an infinitesimal. The latter gives multiplicity.

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