Large fluctuations of time and change of space-time signature

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ABSTRACT

We consider five-dimensional cylindric spacetime V^5 with foliation of codimension 1. The leaves of this foliation are four-dimensional "parallel" universes. The metric of five-dimensional spacetime and induced metrics of four-dimensional universes are flat. The "large" fluctuations of the 5-metric are studied. These fluctuations depend only on the coordinate x^0, and under these fluctuations the curvature of V^5 is not zero. The contribution of the fluctuations in the Feynman path integral over five-dimensional trajectories doesn't change the amplitude of the probability of the real physical four-dimensional universe. Moreover the large fluctuations of 5-metric G_{AB} are large fluctuations for physical four-dimensional universe V^4 and change signature of V^4. The change of the signature from $< + - -- >$ to $< - - -- >$ and inversely occurs in the all 3-dimensional space simultaneously (in absolute time) and can take arbitrarily large period of time.

1 Introduction

It is well-known that spacetime at the Plank's scale can have strong fluctuations of metric. Spacetime at this scale is a "quantum foam" with a high curvature and a veriable topology.

The aim of this work is to construct the metric fluctuations which can exist not only at the Plank's scale but at any distance. Such fluctuations are called "large" [1, 2].

The Kaluza-Klein theory will be used. We find the large fluctuations of 5-metric G_{AB} which are large one for physical four-dimensional global hyperbolic universe V^4 and change signature of V^4. The change of the signature from $< + - -- >$ to $< - - -- >$ and inversely occurs in the all 3-dimensional space simultaneously (in absolute time) and can take arbitrarily large period of time.

2 Basic spacetime

As an example we consider the flat 5-dimensional spacetime with metric

$$dI^2 = G_{AB}dx^A dx^B =$$

$$= (dx^0)^2 - \beta(x^0 - 1)^2(dx^1)^2 - (dx^2)^2 - (dx^3)^2 - (dx^4)^2, \tag{1}$$

where x^0, x^1 is a polar coordinates, x^0 is a radius and x^1 is an angle (see fig.1). Here β is a constant from the interval $(0, 1)$ and $x^0 > 1$. It is evident that this metric has a signature $< + - - - - >$ and

$$V^5 = \mathbb{R}^1 \times S^1 \times \mathbb{R}^3$$
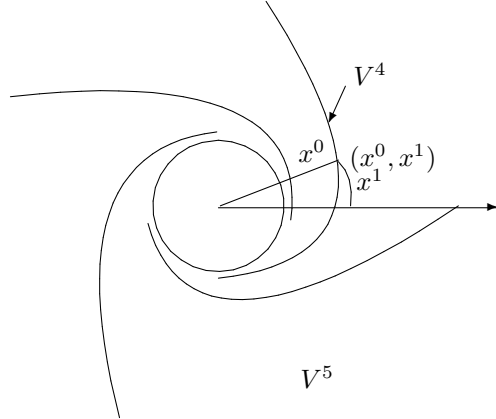
The metric (1) is flat.

Figure 1: Spacetime V^5 with leaf V^4

The physical 4-dimensional universes are defined by equation

$$x^0 = 1 + \alpha \exp(x^1), \qquad (2)$$

where $0 < \alpha < +\infty$ is constant. Each α gives some leaf V_α^4. The equation (2) corresponds to a spiral in the plane (x^0, x^1) (fig.1)

Consider the imbedding $f : V^5 \to \mathbb{R}^6$ of V^5 into $\mathbb{R}^6$

$$\begin{cases} u^0 = x^0 - 1 \\ u^1 = \gamma \cos(x^1) \\ u^2 = \gamma \sin(x^1) \\ u^3 = x^2 \\ u^4 = x^3 \\ u^5 = x^4, \end{cases}$$

Then $f(V^5)$ is cylinder in $\mathbb{R}^6$

$$f(V^5) = \{(u^0, u^1, u^2, u^3, u^4, u^5) \in R^6 : u^0 > 0 \ \& \ (u^1)^2 + (u^2)^2 = \gamma^2\}$$

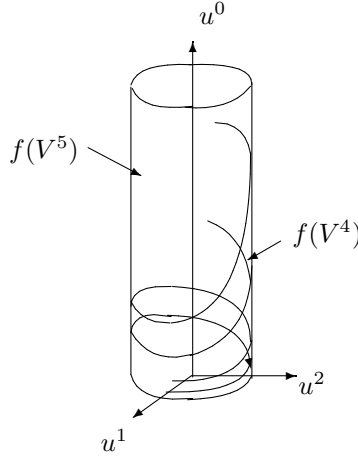where γ is some constant, radius of cylinder (fig.2).

Figure 2: Spacetime $f(V^5)$ with leaf $f(V^4)$

Let (y^0, y^1, y^2, y^3) be a coordinate system of the spacetime V_α^4. These coordinates are accosiated with the coordinates $(x^0, x^1, x^2, x^3, x^4)$ of the 5-dimensional spacetime V^5 by the formulas

$$\begin{cases} x^0 = \alpha \exp(y^0) \\ x^1 = y^0 \\ x^2 = y^1 \\ x^3 = y^2 \\ x^4 = y^3. \end{cases}$$

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Then induced metric of the universe V_α^4

$$g_{ik}^{(4)}(\alpha) = G_{AB} \frac{\partial x^A}{\partial y^i} \frac{\partial x^B}{\partial y^k}$$

in the coordinates y^0, y^1, y^2, y^3 has the form

$$ds_\alpha^2 = \alpha^2(1 - \beta) \exp(2y^0)(dy^0)^2 - (dy^1)^2 - (dy^2)^2 - (dy^3)^2.$$

By making simple transformation

$$y^{0'} = \alpha(1 - \beta)^{1/2} \exp(y^0), \ y^{i'} = y^i, (i = 1, 2, 3)$$

we get

$$ds_\alpha^2 = (dy^{0'})^2 - (dy^{1'})^2 - (dy^{2'})^2 - (dy^{3'})^2.$$

Hence universes V_α^4 are the flat Minkowskian spacetime.

Below for simplicity instead of the cylinder V^5 we consider its factor-space V^5/Γ, where Γ is discrete group $x^0 \to x^0, \ x^1 \to x^1, x^2 \to x^2 + d, x^3 \to x^3 + d, x^4 \to x^4 + d$, which acts on the cylinder (d is an integer).

Topologically the factor-space V^5/Γ is homeomorphic to the space

$$\mathbb{R}^2 \times S^1 \times S^1 \times S^1.$$

It means that the physical 3-dimensional space is 3-dimensional torus $S^1 \times S^1 \times S^1$ and has finite volume.

3 The large fluctuations

The amplitude of probability of transition from universe $V_{\alpha_1}^4$ to universe $V_{\alpha_2}^4$ is equal to

$$< V_{\alpha_1}^4 | V_{\alpha_2}^4 > = \int_{V_{\alpha_1}^4}^{V_{\alpha_2}^4} \exp\left(-\frac{i}{\hbar}S\right) \mathcal{D}[V^5], \tag{3}$$

$$S = \int_{V^5} R\sqrt{det\|G_{AB}\|}d^5x,$$

where Feynman path integral is taken on all five-dimensional trajectories which connect $< V_{\alpha_1}^4, g_{ik}^{(4)}(\alpha_1) >$ and $< V_{\alpha_2}^4, g_{ik}^{(4)}(\alpha_2) >$. Among these trajectories the fluctuations of the 5-dimensional metric (1) can be observed. Let us consider the fluctuations $\tilde{G}_{AB} = G_{AB} + \Delta G_{AB}$ of the 5-dimensional metric (1), for which

$$\tilde{S} = \int_{V^5/\Gamma} \tilde{R}\sqrt{det\|\tilde{G}_{AB}\|}d^5x = 0.$$

We will consider the fluctuations of the form

$$d\tilde{I}^2 = (G_{AB} + \Delta G_{AB})dx^A dx^B =$$

$$= \tilde{G}_{AB}dx^A dx^B =$$
$$= [1 + h(x^0)](dx^0)^2 - \beta(x^0 - 1)^2(dx^1)^2 - (dx^2)^2 - (dx^3)^2 - (dx^4)^2,$$

where $h(x^0)$ is an arbitrary function, such that $h(x^0) > -1$, $h(a) = h(b) = 0$, $a < b$, and $h \equiv 0$ outside of (a, b), i.e. interval (a, b) is an area, where the fluctuations of metric take place (see fig.3).
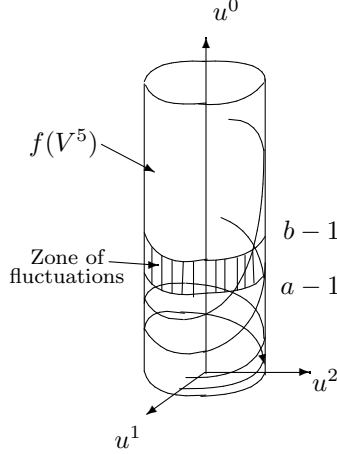

Figure 3: Zone of fluctuations of V^5

We have
$$\tilde{R}_{00} = \frac{1}{2(1 + h)(x^0 - 1)}\frac{dh}{dx^0}, \quad \tilde{R}_{11} = -\frac{\beta(x^0 - 1)}{2(1 + h)^2}\frac{dh}{dx^0},$$

the scalar curvature is
$$\tilde{R} = \frac{1}{(1 + h)^2(x^0 - 1)}\frac{dh}{dx^0},$$

and the determinant of the metrical tensor is
$$det\|\tilde{G}_{AB}\| = (1 + h)\beta(x^0 - 1)^2.$$

So
$$\tilde{S} = \int\limits_{V^5/\Gamma} \beta^{1/2}\frac{1}{(1 + h)^{3/2}}\frac{dh}{dx^0}d^5x =$$

$$= \int\limits_a^b \frac{\beta^{1/2}}{(1 + h)^{3/2}}\frac{dh}{dx^0}dx^0 \int\limits_{\mathbb{R}^3/\Gamma} dx^1 x^2 dx^3.$$

The integration over x^1, x^2, x^3, x^4 gives a constant. Let's consider the integral over x^0

$$\int\limits_a^b \frac{1}{(1 + h)^{3/2}}dh = -\frac{2}{(1 + h)^{1/2}}|_a^b = 0,$$

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because $h(a) = h(b)$. Then the contribution of such fluctuations in the path integral over 5-dimensional trajectories is equal to contribution of basic spacetime V^5. In other words the our basic spacetime can have large fluctuations, which change physical properties of the universe V_α^4.

Indeed, under such fluctuations the 4-dimensional metric $g_{ik}^{(4)}(\alpha)$ takes the form

$$d\tilde{s}^2 = \alpha^2(1 - \beta + h)\exp(2y^0)(dy^0)^2 - (dy^1)^2 - (dy^2)^2 - (dy^3)^2.$$

The geometry of this spacetime is still flat. But the component $g_{00}^{(4)}(\alpha)$ of the metric tensor of the universe V_α^4 has been changed. This change means the change of the gauge on the axis y^0 and, therefore, the change of the speed of light. It occurs instantaneously under all 3-dimensional physical space and, hense, cannot be observed inside the universe V_α^4.

An interesting phenomenon occurs when $h \to 1$. Herewith the signanture of the spacetime V_α^4 is changed from $(+ - - -)$ to $(- - - -)$. The new signature $(- - - -)$ means the cessation of all physical processes [3]. The universe congeals for the arbitrary period of time. This situation can occur in any moment and is not observed.

Note that the stress-energy tensor of our fluctuations

$$\tilde{T}_{22} = \tilde{T}_{33} = \tilde{T}_{44} = -\frac{1}{(1 + h)^2(x^0 - 1)}\frac{dh}{dx^0}$$

is not physical one.

4   Choosing of the function $h(x^0)$

There are many functions $h(x^0)$ which satisfy the condition $h(x^0) > -1$, $h(a) = h(b) = 0$ $(a > b)$. Here are some examples:

$$h(x^0) = C\sin(\frac{2\pi x^0}{a})\sin(\frac{2\pi x^0}{b}),$$

where C is a constant or a function $C(x^0)$ such as $0 < 1 - \beta < C$. Over this function the metric of V^4 changes the signature when $h(x^0) < -1 + \beta$.

$$h(x^0) = (x^0 - a)^2(x^0 - b)^2$$

With this function the metric of V^4 doesn't change the signature. But if $h(x^0)$ is

$$h(x^0) = C(x^0)(x^0 - a)^2(x^0 - b)^2, \quad (-1 < C(x^0)),$$

the metric can change the signature if function $C(x^0)$ is suitable.

References

[1] Modanese,G. Virtual dipoles and large fluctuations in quantum gravity, *Phys. Letters.* **B 460**, 276 (1999) .

[2] Modanese,G. Large "Dipolar" Vacuum Fluctuations in quantum gravity, Los Alamos E-Print gr-qc/0005009.

[3] Sakharov, A.D. Cosmological transitions with change of signature. *ZhETF.* **87** (1984).