assumed that background metric tensor γ_{ik} describes a flat space-time [2-5]. We assume that γ_{ik} is a dynamical variable which shall be determined from the extremity condition of the total action $S = S_\varphi + S_g + S_\gamma$, where S_φ is the action of the scalar field φ [1] and

$$S_\gamma = -\frac{1}{2}\beta_0 \int g^{ik}\check{R}_{ik}\sqrt{-g}d^4x, \tag{2}$$

is the action of a free $\gamma_{ik}, \beta_0 = 1/8\pi G, \check{R}_{ik}$ is the Ricci tensor formed from γ_{ik} and now $\beta \neq 1/8\pi G$ is a new constant. The assumption that γ_{ik} is a dynamical variable allows to introduce an additional tensor field ψ_{ik} (see below). S_γ is the simplest expression that depends on $\check{R}_{ik}$ and similar to (1) is invariant under the scale transformation $\gamma_{ik} \rightarrow a\gamma_{ik}$, where a is an arbitrary constant.

Varying S with respect to g_{ik} we come to the gravitational field equations

$$\beta R_{ik} + (\beta_0 - \beta)\check{R}_{ik} = T_{ik} - \frac{1}{2}g_{ik}T, \qquad T = g^{ik}T_{ik}, \tag{3}$$

where R_{ik} is the Ricci tensor formed from g_{ik} and T_{ik} is the energy-momentum tensor of the scalar field φ. Then varying S with respect to γ_{ik} we have

$$(\beta - \beta_0)[\sqrt{g/\gamma}(\gamma^{ik}g^{nm} + \gamma^{nm}g^{ik} - \gamma^{in}g^{km} - \gamma^{kn}g^{im})]_{:nm} = 0, \tag{3}$$

where : denotes a covariant derivative with respect to γ_{ik} and $\gamma^{in}\gamma_{nk} = \delta^i_k$. In case of $\beta = \beta_0$ (4) reduces to an identity and (3) transforms to the Einstein equations for φ with $g_{ik} = g_{ik}(\varphi)$. In case of $\beta \neq \beta_0$ (4) transforms to the field equations for γ_{ik}. These equations have a partial solution $\gamma_{ik} = ag_{ik}$ for which again $g_{ik} = g_{ik}(\varphi)$ as it follows from (3). In general case it may be introduced a tensor field ψ_{ik} by the relation $\gamma_{ik} = a(g_{ik} + \psi_{ik})$. Using this definition of ψ_{ik} the total action may be presented in the following form

$$S = S_\varphi - \frac{1}{2}(\beta - \beta_0)\int g^{ik}(\Delta^l_{in}\Delta^n_{kl} - \Delta^l_{ik}\Delta^n_{ln})\sqrt{-g}d^4x - \frac{1}{2}\beta_0\int R\sqrt{-g}d^4x + \sigma, \tag{5}$$

where $\Delta^l_{ik} = 0.5\tilde{\gamma}^{ln}(\psi_{ik;n} - \psi_{ni;k} - \psi_{nk;i}), \tilde{\gamma}^{in}(g_{nk} + \psi_{nk}) = \delta^i_k, R = g^{ik}R_{ik}$ and ; is a covariant derivative with respect to g_{ik}, σ is an integral of a 4-divergence that may be omitted. Exp.(5) is the action of GR for the system of self-gravitating scalar φ and nonlinear tensor ψ_{ik} fields. Eqns.(3) also may be presented in the form similar to the usual Einstein equations if we introduce the following energy-momentum tensor

$$\Pi_{ik} = (\beta - \beta_0)[\check{R}_{ik} - R_{ik} - \frac{1}{2}g_{ik}g^{nm}(\check{R}_{nm} - R_{nm})] \tag{6}$$

for ψ_{ik}. From (4) it follows that the weak tensor field ψ_{ik} and the weak gravitational wave propagating in the curved space-time are determined by the same equations.

Eqns.(3), (4) and the field equation for φ allow to determine ψ_{ik}, g_{ik} and φ. Among the numerous solutions there will be also solutions with effective negative pressure $p_{eff} = -\rho$ presenting a special cosmological interest [1] (ρ is the energy density of the scalar and the tensor fields).

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The Mean Number of 4-Wormholes in the Universe

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The 4-dimensional wormholes are appeared as result of evolution of massive stars [1]. It can calculate their mean number if to consider the stochastic process $x = \{x_t : t \in [0, \infty)\}$, where t is not time (but $t = x^6$ in 6-dimensional theory of gravitation that it is used), in some probability space $< \Omega, S, \mathbf{P} >$ with phase space $< \mathcal{V}, \mathcal{T} >$. Here $\mathcal{V}$ is the set of all different universes that are formed from the Lorentz manifold W^4 by means of the attaching of 4-dimensional handles (4- wormholes). The topology $\mathcal{T}$ is described in [2].

Let $g : \mathcal{V} \to \mathbb{Z} \subset \mathbb{R}$ be a function such that $g(v)$ is the number of 4-wormholes of universe v. Suppose that every $v \in \mathcal{V}$ has a neighborhood which does not contain $w \in \mathcal{V}$ with $g(w) \neq g(v)$. Then g is continuous and one can consider the stochastic process $g = \{g \circ x_t : t \in [0, \infty)\}$ with number phase space.

If process $g \circ x$ is stationary measurable one and $M\{g \circ x_0\} < \infty$ then with probability 1

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t g \circ x_s(\omega) ds = M\{g \circ x_0 \mid \mathcal{L}\},$$

where $\mathcal{L}$ is σ-algebra of invariant ω-sets defined by means of process $g \circ x$. In a number of cases the conditional mean $M\{g \circ x_0\}$ (that is the same for all t) is mean number of 4-wormholes in the Universe.

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Qualitative Tilted Homogeneous Cosmologies

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We discuss spatially homogeneous cosmological models of Bianchi types II - VII admitting a perfect fluid source which does **not** flow orthogonal to the hypersurfaces of homogeneity. These models are classified into five sub-classes according to the action of the Abelian G_2 subgroup which they admit. Evidence is provided to support the claims that:

a) Whimper singularities are not generic,

b) Only two of the five sub-classes admit chaotic behaviour.

Periodicity, Compactification and Self-Similarity in Bianchi-IX Models

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Studies of the dynamical behaviour of Bianchi-IX cosmologies near their spacetime singularities have included methods where the original set of ordinary differential equations (derived from the Einstein equations) can be approximated by discrete iterative maps that describe the dynamics as transitions from one Kasner solution to another. These discrete maps (of the form $x_{n+1} = F(x_n)$ where n labels the iteration number) have been shown to be chaotic in the sense that they have at least one positive Lyapunov exponent which measures the system's sensitivity on the precision with which one specifies the initial conditions. While much effort has focused on proving that the discrete maps can provide an accurate description of the full continuous time dynamics particularly in the regime where the maps themselves are chaotic, little work has been applied to understanding non-chaotic solutions to the discrete maps and their relationship to the full dynamics.

In this work periodic solutions are found to the Bogoyavlensky map [2] which represents an approximation to the dynamics using the orthonormal tetrad method of Ellis and MacCallum [3]. The map is given as

$$x_{n+1} = \cos^{-1}\left(\frac{4 - 5\cos x_n}{5 - 4\cos x_n}\right)$$

on the interval $0 \leq x_n \leq \frac{\pi}{3}$. The periodic transitions allow one to specify the dynamical shear components which act as initial conditions for the full set of Einstein equations.

The periodic solutions to the map lead to discrete self-similar solutions to the full set of ODE's describing the Bianchi-IX dynamics where there is a linear scaling in the logarithmic time coordinate. Rescaling the dynamical variables recovers the periodicity of the Bogovavlensky map in the continuous system. The rescaling also provides a compactification of the phase space variables so that one can use the singularity avoiding logarithmic time coordinate along with dynamical variables that remain finite for all time. (A compact phase space is necessary in order to discuss the possibility of chaotic behaviour in nonlinear dynamical systems.)

Using the Belinskii, Khalatnikov, Lifshitz (or BKL) [4] approach to Bianchi-IX cosmologies one can derive a discrete one-dimensional map between so-called "Kasner-epochs" and in the appropriate variables this can be written