STOCHASTIC EVOLUTION OF THE TOPOLOGY
AND GEOMETRY OF SPACE–TIME AND THE
SIX-DIMENSIONAL THEORY OF GRAVITATION

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A theory of random processes with a suitable phase space is used to predict the variation of topology and geometry, and thus also of gravitational fields in vase regions of space–time, that is, on the macrolevel.

During the evolution of massive stars under certain circumstances, topological rearrangements of space–time take place [1, 2]. The topology of space–time becomes multiply connected. Therefore, in studies where it is no longer allowed to neglect the variation of topology, and thus of geometry at stellar distances, it is necessary to employ a mathematical model of space–time which adequately represents the situation which has actually developed. However, the evolution of stars in the Galaxy, or in the Universe on a large scale, is random, so that the model selected will require periodic correction. From the point of view of the general theory of relatively, on the other hand, space–time, in which world history unrolls, is absolute [3] (in contrast to a relative dynamically variable space and time), given once and for all, and the substitution of a space–time model essentially means changing to a theory allowing a dynamic variation of space–time itself. Since space–time loses its absolute character, it becomes possible to affect its topological, metrical, and causal structure. The freedom of action thereby allowed mathematically can be formulated as a transition to a more than four-dimensional model describing the Universe. The new measurement (having six dimensions in this article) represents, to some extent, a freedom of choice of actions, granted by nature to the researcher, or free will, if we allow ourselves to use a term more acceptable for a philosopher than for a mathematician.

1. RANDOM VARIATIONS OF SPACE–TIME TOPOLOGY

In current relativistic cosmology the model of space–time is a four-dimensional Lorentz manifold M^4, possessing four-dimensional wormholes, that is, M^4 is a manifold obtained from some smooth manifold W^4 by attaching to it a 4-handle $C = S^3 \times D^1$ via the imbedding $f: S^0 \times D^3 \to W^4$ (see [4]). The manifold obtained thereby is designated as $M^4 = W^4[f]$. Let us replace model $W^4[f_1][f_2]...[f_n]$ with n 4-handles by a model with m 4-handles $W^4[g_1][g_2]...[g_m]$. This is done because, as noted above, the previous model describes poorly the actual situation which had developed after new knowledge about the Universe was obtained. The appearance of a new 4-handle is a certain, not always realized, stage in the evolution of a specific star, and this event is naturally taken to be random. Actually, in order for a topological rearrangement of space–time to take place, associated with certain physical processes occurring in the interior of the star, a whole set of conditions must be satisfied [1, 2], each of which is influenced by a large number of random factor.

Consequently, we can define a random process $x = \{x_t: t \in T\}$ in some probability space $<\Omega, S, P>$ and a phase measurable topological space $<V, I>$, where V is the set of all different manifolds of form $M_t^4 = W^4\{f_{t1}][f_{t2}]...[f_{tn(t)}]$ (the images of the pair of disks $S^0 \times D^3$ for different imbeddings f_{ti} are taken to be different; this makes the above-described

topology T_R a Hausdorff topology). In order to specify topology T_R in V, we assume that each space–time M_t^4 is imbedded as a submanifold into 5-dimensional Lorentz manifold M_t^5, and that the family $\{M_t^5: t \in T\}$ is imbedded into a 6-dimensional manifold M^6. The fifth dimension for us represents a purely mathematical device (as will be evident from the following), needed to embody the planned idea. This means that the meaning of the 5th coordinate cannot be ascertained from the given work. Nevertheless, for a complete mathematical description of the stage of the stochastic evolution of the topology and geometry (at distances comparable to interstellar distances) of a four-dimensional Universe, the 6-dimensional variant of the Kaluza–Klein theory is quite suitable.

2. TOPOLOGY T_R IN PHASE SPACE V

Let us consider an element $M_t^4 = W^4[f_{t1}][f_{t2}]...[f_{tn(t)}]$ of set V, a future point of the phase topological space $<V, T_R>$, where $f_{ti}:S^0 \times D^3 \to W^4$ is an imbedding (i = 1,..., n(t)), t = x^6. We assume that $B_{ti}^1 \cup B_{ti}^2 = f_{ti}(S^0 \times D^3)$, that always $B_{ti}^1 \cup B_{ti}^2 = \varnothing$, and that cylinder $C_{ti} = S^3 \times D^1 \subseteq M_t^5$ joins ∂B_{ti}^1 with ∂B_{ti}^2 in M_t^5. Moreover, let us identify the 4-handle $[f_{ti}]$ with the set of three $<B_{ti}^1, B_{ti}^2, C_{ti}>$. We know by definition that the neighborhood of the point (4-handle)

$$[f_{t1}][f_{t2}]...[f_{tn(t)}] = \bigcup_{i=1}^{n(t)} <B_{ti}^1, B_{ti}^2, C_{ti}>$$

is the set

$$[\hat{f}_1][\hat{f}_2]...[\hat{f}_{n(t)}] = \bigcup_{i=1}^{n,t} <\hat{B}_i^1, \hat{B}_i^2, \hat{C}_i> =$$

$$= \bigcup_{i=1}^{n(t)} \{[f_i] = <B_i^1, B_i^2, C_i>: B_{ti}^1 \subseteq \operatorname{int} B_i^1 \subseteq \operatorname{int} \hat{B}_i^1, B_{ti}^2 \subseteq \operatorname{int} B_i^2 \subseteq \operatorname{int} \hat{B}_i^2,$$

C_i keeps C_{ti} inside it and itself lies inside of $\hat{C}_i\}$, where all $\hat{B}_i^j$ pairwise do not intersect, $\hat{C}_i = S^3 \times D^1 \subseteq M_t^5$ joints $\partial \hat{B}_i^1$ with $\partial \hat{B}_i^2$ in M_t^5.

Let us assume also that the "empty" 4-handle [ ] = $<B^1, B^2, \varnothing>$ and the "4-handle-point" [pt] = $<B^1, B^2, \mathrm{pt}>$ with B^2 joined at a point, as equivalent points of space V. The first models the time when the given region B^1 of space–time "intended" to begin the topological rearrangement and to "unite" with region B^2, while the second corresponds to the critical time when regions B^1 and B^2, having "rushed toward" one another, first touch for the time being at a single point. Symbolically this can be portrayed as a process (in "time" t = x^6) of creation of a 4-handle: [ ] $\to$ [f]. In equal measure we can refer to the opposite process, disappearance of a 4-handle: [f] $\to$ [pt] $\to$ [ ].

The introduced topology is Hausdorff locally compact regular and with a denumerable base, that is, $<V, T_R>$ is metrizable.

Therefore, since from the viewpoint of the multidimensional theory of gravitation the stochastic evolution (in the sixth dimension) of the topology of space–time, because of various processes with stellar matter, entails the stochastic evolution of a Lorentzian geometry, therefore it is quite possible to use the theory of random processes with a suitable phase space to predict the variation of the geometry and, hence, of the gravitational fields in vast regions of space–time, that is, on the macrolevel.

3. FINAL EVOLUTIONARY STAGE OF THE TOPOLOGY OF THE UNIVERSE

Although the selection of some model of space–time is the task of the researcher, he still with his "freedom" of action tries to find a model which adequately portrays the accumulated data and theory concerning the Universe. Consequently, the state to which a process s = $\{x_t: tt \in T\}$ converges in "time" t as t $\to \infty$ may give us in some sense an ideal model of space–time. Since a stochastic process is involved here, we are referring to a state in the probability-theory sense only.

Let x_t be a uniform Markov process with transition probabilities $P(t, v, A)$, $t \geq 0$, $v \in V$, $A \in I$ being the σ algebra of Borel sets. We assume that for any $\varepsilon > 0$ there exist: a finite measure $m = m(A)$ in phase space $<V, I>$, a set $A_0 \in I$, and positive number t_0, δ such that for any initial distribution P_0, $t \geq t_0$ and $A \subseteq A_0$

$$P(t_0, v, A) \geqslant \delta \cdot m(A) \quad (v \in A_0),$$
$$P_t(A) = \int_V P(t, v, A)\, P_0(dv) \leqslant m(A) + \varepsilon,$$
$$P_t(A_0) \geqslant 1 - \varepsilon.$$

Then [5, p. 685; 6, p. 446] there exists a single invariant probability distribution $\hat{\mathbf{P}}$ which is "ergodic," that is, $d(\hat{\mathbf{P}}) \to 0$ as $t \to \infty$, no matter what the initial distribution $\mathbf{P}_0$ may be. Here

$$d(\mathbf{Q}, \mathbf{P}) = \int_V |\mathbf{Q}(dv) - \mathbf{P}(dv)|$$

is the metric in the set of all distributions in V, being a complete metric. Therefore, $d(\mathbf{P}_{t_n}, \mathbf{P}_{t_m}) \to 0$, no matter what $\mathbf{P}_0$ and $t_n \to \infty$ may be.

Therefore, with certain restrictions, independently of the initial probability distribution in the space of possible models of the space–time of the Universe, while improving the accuracy of the topology and geometry, the investigator inevitably selects the only possible most probable classes of models of space–time. At the same time, this means that the most probable final topological-geometrical stages (at interstellar distances) of the evolving Universe are ascertained.

4. AVERAGE NUMBER OF FOUR-DIMENSIONAL WORMHOLES IN THE UNIVERSE

Assume that g: $V \to \mathbf{Z}$, where $\mathbf{Z}$ is the set of whole numbers, is a function the values of which g(v) are the number of 4-handles for a model v. Assume, too, that the set of 4-handles $\{[f_{t_i}]\}$ is such that, being considered all at once in W^4, nowhere in W^4 do they become condensed. More precisely, at each point v of V, there exists a neighborhood not containing points w having a number of 4-handles different from v. This means that there is a lower limit to the spatial dimensions of the regions of space–time in which 4-handles originate, that is, we consider only macroscopic 4-dimensional wormholes. Then function g becomes continuous, and we can examine the random process g $\circ$ x = $\{g \circ x_t : t \in T\}$ with a numerical phase space. If process g $\circ$ x is measured under steady-state conditions with $\mathbf{M}\{g \circ x_0\} < \infty$, than with probability of 1

$$\frac{1}{t} \int_0^t (g \circ x_s)(\omega)\, ds \underset{t \to \infty}{\to} M\{g \cdot x_0 \,|\, L\},$$

where L is the σ algebra of the invariant ω sets defined by process g $\circ$ x [5, p. 527]. In some cases the conditional average $\mathbf{M}\{g \circ x_0 \,|\, L\}$ is replaced by the mean value of the 4-handles $\mathbf{M}\{g \circ x_0\}$ (the same for each "time" t), which is the average number of 4-dimensional wormholes in space–time.

Assumptions concerning the uniformly or stationary of the considered stochastic processes are associated to some degree with the assumption of stationarity of the stellar evolutionary processes in the Universe, but even more with the assumption of stationarity of the modeling of the topology of space–time, that is, with stationarity of our knowledge of the Universe.

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